Co-ordinate Systems :

Basically, there are three co-ordinate systems are involved in electromagnetics

1. Cartesian Co-ordinate system.
2. Cylindrical co-ordinate system.
3. Spherical Co-ordinate system.

Rectangular Co-ordinates (x, y, z).

A rectangular or Cartesian co-ordinate system has three mutually perpendicular axes called x, y and z axes.

Here $-\infty < x < \infty$
$-\infty < y < \infty$.
$-\infty < z < \infty$.

A vector at the origin of rectangular co-ordinate system may be resolved into three component vector

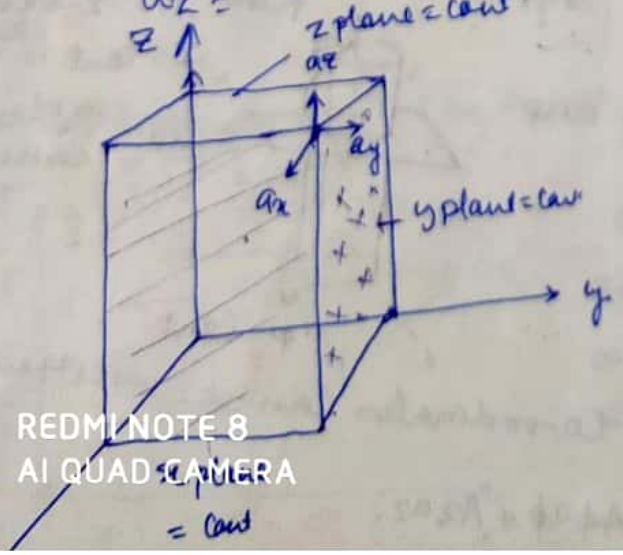
$$A = A_x a_x + A_y a_y + A_z a_z.$$

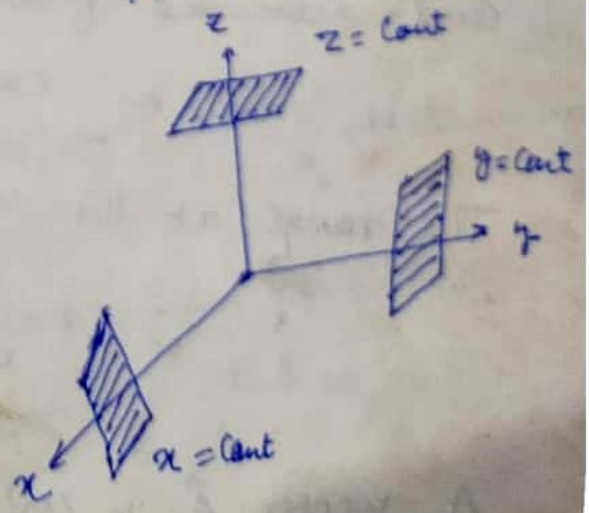
$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}.$$

a_x = unit vector in the x dir^n = $\frac{A_x}{|A|}$.

a_y = unit vector in the y dir^n = $\frac{A_y}{|A|}$.

a_z = unit vector in the z dir^n = $\frac{A_z}{|A|}$





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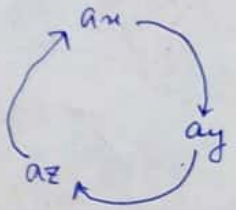
Cross Product – The Cross Product of two vector $A \times B$

$$A \times B = |A||B| \sin\theta_{AB}.$$

If $A = (A_x, A_y, A_z)$, and $B = (B_x, B_y, B_z)$.

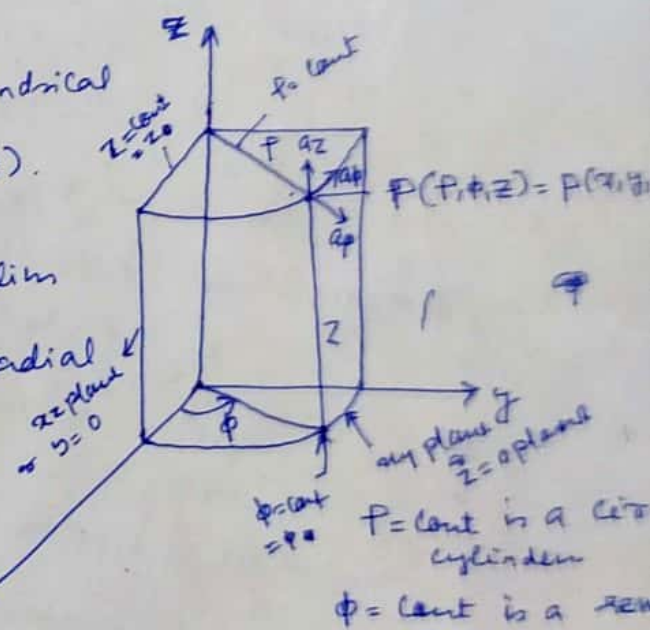
$$A \times B = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y) a_x + (A_z B_x - A_x B_z) a_y + (A_x B_y - A_y B_x) a_z$$



$$\left.\begin{aligned} a_x \times a_y &= a_z \\ a_y \times a_z &= a_x \\ a_z \times a_x &= a_y \end{aligned}\right\} \text{cyclic permutation}$$

Cylindrical Co-ordinates (ρ, ϕ, z):

Let us consider the point P in cylindrical co-ordinates is represented as (ρ, ϕ, z).



Here the space variable ρ is the radius of the cylinder through P or the radial distance from z-axis.

ϕ is called the azimutal angle. and is measured from x-axis in the xy plane

z is same as cartesian System.

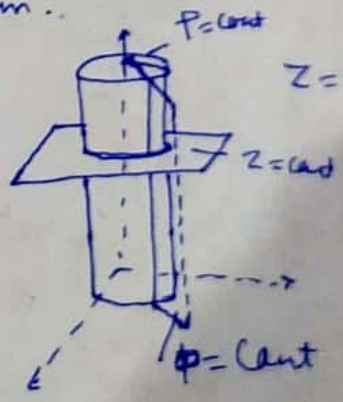
ρ = const is a cir cylinder

ϕ = const is a sem infinite plan its edge along z-axis.

z = const is the same t plane as in Cartesian co-

The range of the variables are

$$0 \leq \rho < \infty$$
$$0 \leq \phi < 2\pi$$
$$-\infty < z < \infty.$$



A vector $\vec{A}$ is cylindrical co-ordinates can be written as (A_ρ, A_ϕ, A_z)

$$\vec{A} = A_\rho a_\rho + A_\phi a_\phi + A_z a_z.$$

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Since the unit vector a_ρ, a_ϕ and a_z are mutually perpendicular because our co-ordinate system is orthogonal.

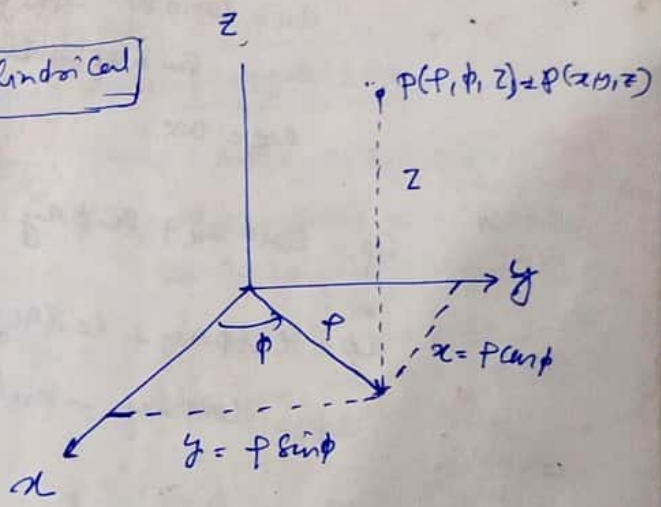
Relⁿ betⁿ dot and cross product of unit vector in cylindrical co-ordinate system

$$a_\rho \cdot a_\rho = a_\phi \cdot a_\phi = a_z \cdot a_z = 1.$$
$$a_\rho \cdot a_\phi = a_\phi \cdot a_z = a_z \cdot a_\rho = 0.$$

and

$$\left.\begin{aligned} a_\rho \times a_\phi &= a_z. \\ a_\phi \times a_z &= a_\rho \\ a_z \times a_\rho &= a_\phi \end{aligned}\right\} \text{ It can be obtain in cyclic permutation.}$$

Relⁿ betⁿ Cartesian co-ordinate and cylindrical co-ordinate :

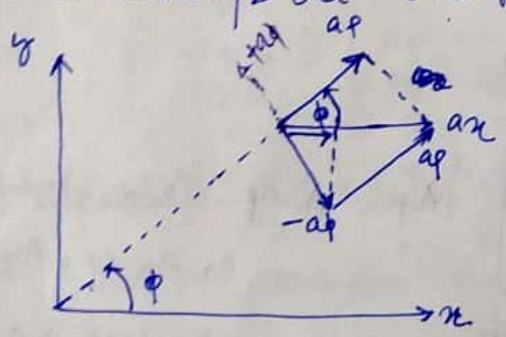


$$x = \rho \cos\phi$$
$$y = \rho \sin\phi$$
$$z = z$$

$$\boxed{\begin{aligned} \rho &= \sqrt{x^2+y^2} \quad \rho \geq 0 \\ \phi &= \tan^{-1}(y/x) \\ z &= z. \end{aligned}}$$

The above eqⁿ is transforming a point from Cartesian (x, y, z) to cylindrical (ρ, ϕ, z).

The relationships betⁿ (a_x, a_y, a_z) and (a_ρ, a_ϕ, a_z) are obtain as shown in fig.



$$a_x = \cos\phi\, a_\rho + \cos(90-\phi)(-a_\phi)$$
$$a_x = \cos\phi\, a_\rho - \sin\phi\, a_\phi$$

Next vector transformation in cylindrical components of a_x,

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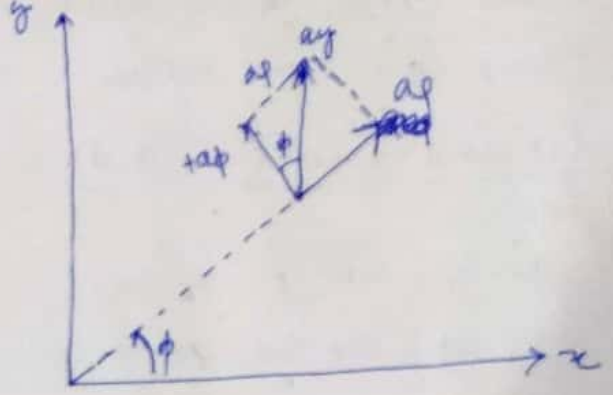
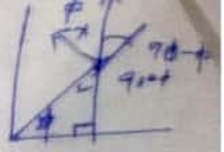
$a_y = \cos(90-\phi)\, a_\rho + \cos\phi\, a_\phi$

$a_y = \sin\phi\, a_\rho + \cos\phi\, a_\phi$



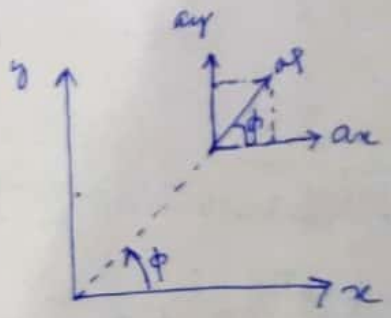


Cylindrical Component of a_z is a_z.

$\therefore$ $a_x = \cos\phi\, a_\rho - \sin\phi\, a_\phi$

$a_y = \sin\phi\, a_\rho + \cos\phi\, a_\phi$

$a_z = a_z$.

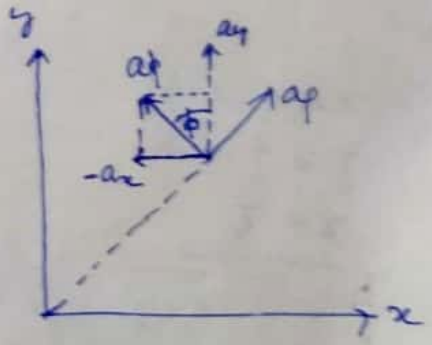


$a_\rho = \cos\phi\, a_x + \sin\phi\, a_y$

$a_\phi = \cos\phi\, a_y + \cos(90-\phi)(-a_x)$

$a_\phi = \cos\phi\, a_y - \sin\phi\, a_x$

$a_z = a_z$.



$\vec{A} = A_x a_x + A_y a_y + A_z a_z$

~~Unit vector Transform in cylindrical component of a_y.~~

$\vec{A} = A_\rho a_\rho + A_\phi a_\phi + A_z a_z$.

$\vec{A}\cdot a_\rho = (A_\rho a_\rho + A_\phi a_\phi + A_z a_z)\cdot a_\rho = A_\rho\, a_\rho\cdot a_\rho + A_\phi\, a_\phi\cdot a_\rho + A_z\, a_z\cdot a_\rho$

$= A_\rho$.

$A_\rho = \vec{A}\cdot a_\rho$.

$\vec{A}\cdot a_\phi = A_\phi$, $\vec{A}\cdot a_z = A_z$.

Hence $A_\rho = \vec{A}\cdot a_\rho$
$A_\phi = \vec{A}\cdot a_z$
$A_z = \vec{A}\cdot a_z$

$A_\rho = \vec{A}\cdot a_\rho = (A_x a_x + A_y a_y + A_z a_z)\cdot a_\rho$

$= A_x a_x\cdot a_\rho + A_y a_y\cdot a_\rho + A_z a_z\cdot a_\rho$

$= A_x \cos\phi + A_y \sin\phi + 0$

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$$A_\phi = \vec{A}\cdot a_\phi = (A_x a_x + A_y a_y + A_z a_z)\cdot a_\phi$$
$$= A_x\, a_x\cdot a_\phi + A_y a_y\cdot a_\phi + A_z a_z\cdot a_\phi$$
$$= A_x(-\sin\phi) + A_y \cos\phi + 0$$

$$A_z = \vec{A}\cdot a_z = (A_x a_x + A_y a_y + A_z a_z)\cdot a_z$$
$$= A_x\, 0 + A_y\, 0 + A_z\cdot 1$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\boxed{\vec{A} = (A_x\cos\phi + A_y\sin\phi)\, a_\rho + (-A_x\sin\phi + A_y\cos\phi)\, a_\phi + A_z a_z}$$

Similarly $A_x = \vec{A}\cdot a_x$ $\quad A_x = (A_\rho a_\rho + A_\phi a_\phi + A_z a_z)\cdot a_x$

$A_y = \vec{A}\cdot a_y$ $\quad = A_\rho\, a_\rho\cdot a_x + A_\phi\, a_\phi\cdot a_x + A_z\, a_z\cdot a_x$

$A_z = \vec{A}\cdot a_z$ $\quad = A_\rho\cos\phi - A_\phi\sin\phi + 0\cdot A$

$A_y = A_\rho\sin\phi + A_\phi\cos\phi + 0\cdot A$

$A_z = A_\rho\, 0 + A_\phi\cdot 0 + 1\, A_z$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

<u>Alternative</u>

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} a_x\cdot a_\rho & a_x\cdot a_\phi & a_x\cdot a_z \\ a_y\cdot a_\rho & a_y\cdot a_\phi & a_y\cdot a_z \\ a_z\cdot a_\rho & a_z\cdot a_\phi & a_z\cdot a_z \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} a_\rho\cdot a_x & a_\rho\cdot a_y & a_\rho\cdot a_z \\ a_\phi\cdot a_x & a_\phi\cdot a_y & a_\phi\cdot a_z \\ a_z\cdot a_x & a_z\cdot a_y & a_z\cdot a_z \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

<u>Spherical co-ordinate</u> : (r, θ, ϕ)

Let us consider a point P can be represented as (r, θ, ϕ)

Here r is defined as the distance from the origin to

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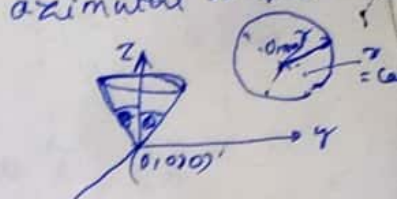
θ is called co-latitude angle, it is the angle betn the z-axis and the position vector of P. (* θ co-ordinate represents the angle made by co with z-axis. The θ-co-ordinate always positive)

ϕ is measured from the x-axis (the same azimutal angle in cylindrical co-ordinates).



The range of the variables are

$$0 \le r < \infty$$
$$0 \le \theta \le \pi$$
$$0 \le \phi < 2\pi$$

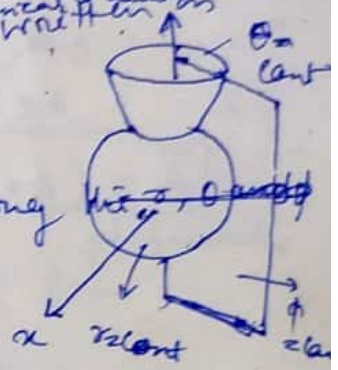
r = cont is a sphere with its centre at origin

θ = cont is a circular cone with z-axis as its axis and origin as its vertex

ϕ = cont is the semi infinite plane as in cylindrical co-ordinates

A vector $\vec{A}$ in spherical co-ordinates (A_r, A_θ, A_ϕ) $\quad \vec{A} = A_r a_r + A_\theta a_\theta + A_\phi a_\phi$.

where a_r, a_θ, a_ϕ are the unit vectors along the r, θ, ϕ dirn

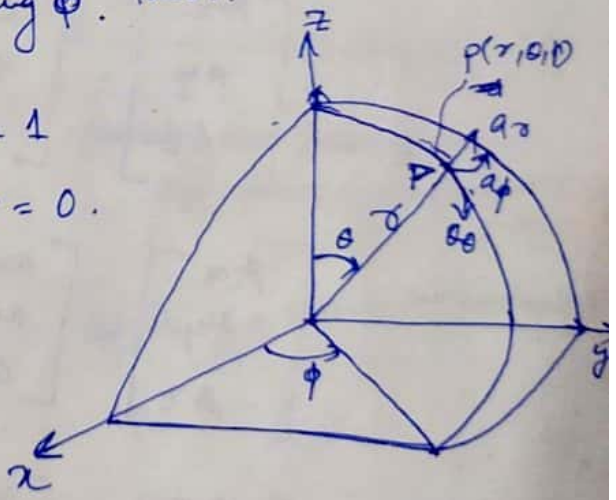


$$|A| = \sqrt{A_r^2 + A_\theta^2 + A_\phi^2}$$

The unit vectors a_r, a_θ and a_ϕ are mutually orthogonal, a_r being along the radius, a_θ is the dirn of increasing θ. and a_ϕ in the dirn of increasing ϕ. Thus.

$$a_r \cdot a_r = a_\theta \cdot a_\theta = a_\phi \cdot a_\phi = 1$$
$$a_r \cdot a_\theta = a_\theta \cdot a_\phi = a_\phi \cdot a_r = 0.$$

$$a_r \times a_\theta = a_\phi$$
$$a_\phi \times a_r = a_\theta$$
$$a_\theta \times a_\phi = a_r$$



The space variable (x, y, z) in Cartesian co-ordinate can be related to variables (r, θ, ϕ) of spherical co-ordinate

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$

$$r = \sqrt{x^2 + y^2 + z^2} \qquad r \geq 0$$
$$\theta = \cos^{-1} \frac{z}{r} = \cos^{-1} \frac{z}{\quad}$$

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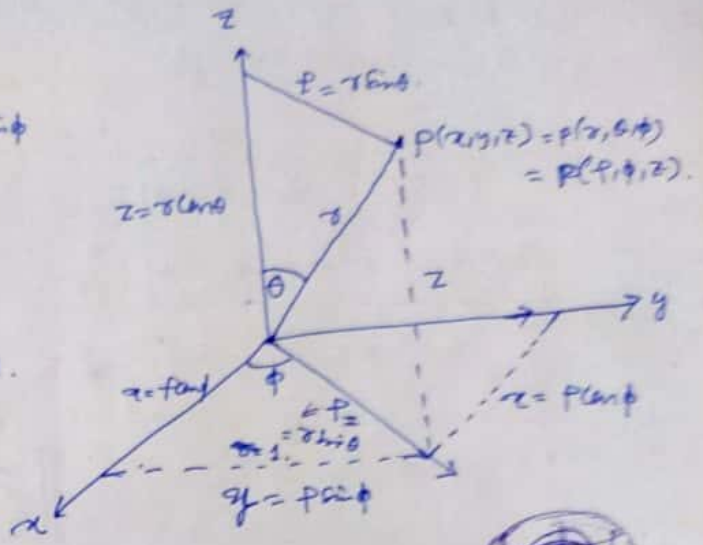
$x = \rho \cos\phi$

$x = r \sin\theta \cos\phi$

$y = \rho \sin\phi = r \sin\theta \sin\phi$

$z = r \cos\theta$.



$\vec{A} = A_x a_x + A_y a_y + A_z a_z$.

$\vec{A} = A_r a_r + A_\theta a_\theta + A_\phi a_\phi$

$\vec{A} \cdot a_r = (A_r a_r + A_\theta a_\theta + A_\phi a_\phi) \cdot a_r$

$= A_r$

$A_r = \vec{A} \cdot a_r$

Similarly $A_\theta = \vec{A} \cdot a_\theta$, $A_\phi = \vec{A} \cdot a_\phi$.

$A_r = \vec{A} \cdot a_r = (A_x a_x + A_y a_y + A_z a_z) \cdot a_r$

$= A_x\, a_x \cdot a_r + A_y\, a_y \cdot a_r + A_z\, a_z \cdot a_r$

$A_\theta = \vec{A} \cdot a_\theta = (A_x a_x + A_y a_y + A_z a_z) \cdot a_\theta$

$= A_x\, a_x \cdot a_\theta + A_y\, a_y \cdot a_\theta + A_z\, a_z \cdot a_\theta$.

$A_\phi = \vec{A} \cdot a_\phi = (A_x a_x + A_y a_y + A_z a_z) \cdot a_\phi$

$= A_x\, a_x \cdot a_\phi + A_y\, a_y \cdot a_\phi + A_z\, a_z \cdot a_\phi$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} a_r \cdot a_x & a_r \cdot a_y & a_r \cdot a_z \\ a_\theta \cdot a_x & a_\theta \cdot a_y & a_\theta \cdot a_z \\ a_\phi \cdot a_x & a_\phi \cdot a_y & a_\phi \cdot a_z \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} a_x \cdot a_r & a_x \cdot a_\theta & a_x \cdot a_\phi \\ a_y \cdot a_r & a_y \cdot a_\theta & a_y \cdot a_\phi \\ a_z \cdot a_r & a_z \cdot a_\theta & a_z \cdot a_\phi \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

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$a_r \cdot$	$\cos\theta$	$-\sin\theta$	0	$a_\phi \cdot$	$\cos\theta$	$-\sin\theta$	0

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\cdot\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

~~The converse transformation~~

Similarly we obtain

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ +\cos\theta\cos\phi & \cos\theta\cdot\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Express the vector $B = \frac{10}{r} a_x + x\cos\theta\, a_\theta + a_\phi$ in Cartesian and Cylindrical

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Differential Length, Area and volume:

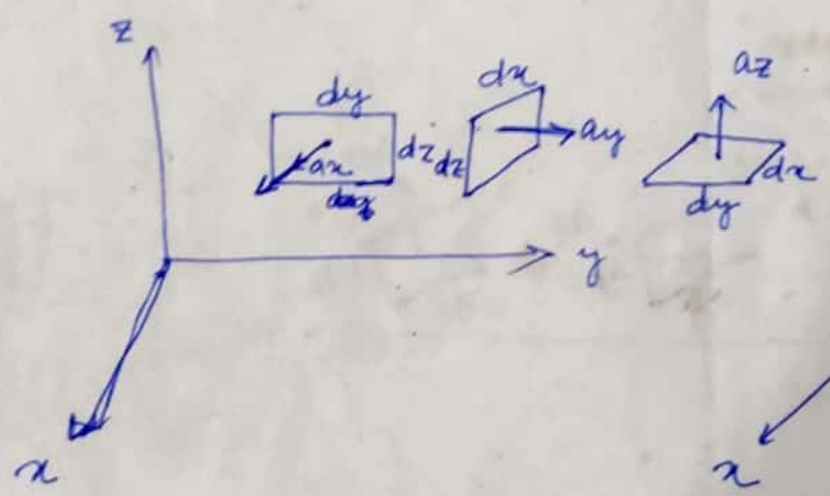
(a) Cartesian co-ordinates:

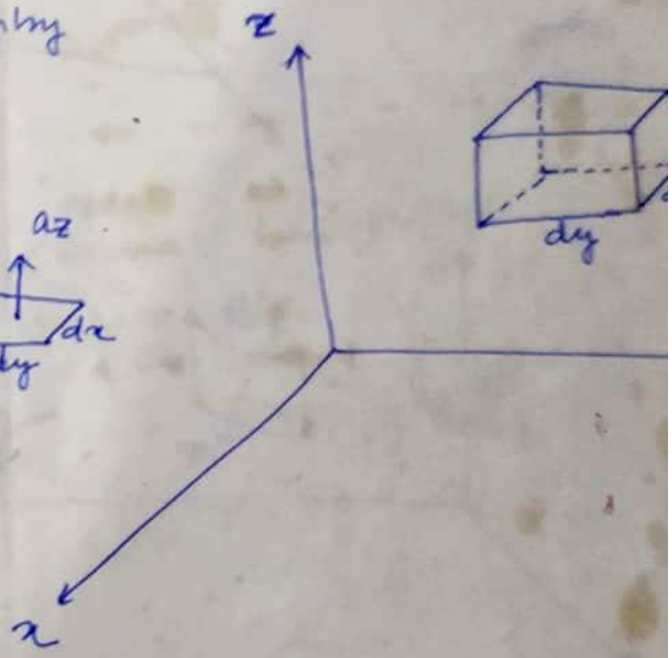
The enerienental displacement is given by

$$(dL) = dx\,a_x + dy\,a_y + dz\,a_z.$$

Differential normal area is given by $(ds) = dy\,dz\,a_x$
$dx\,dz\,a_y$
$dx\,dy\,a_z$

Differential volume is given by
$$dv = dx\,dy\,dz.$$





(b) Cylindrical Co-ordinates:

The differential displacement is given by
$$(dL) = d\rho\,a_\rho + \rho\,d\phi\,a_\phi + dz\,a_z$$

The differential normal area is given by
$$(ds) = \rho\,d\phi\,dz\,a_\rho$$
$$d\rho\,dz\,a_\phi$$
$$\rho\,d\phi\,d\rho\,a_z$$

Differential volume is given by $dv = \rho\,d\rho\,d\phi\,dz.$

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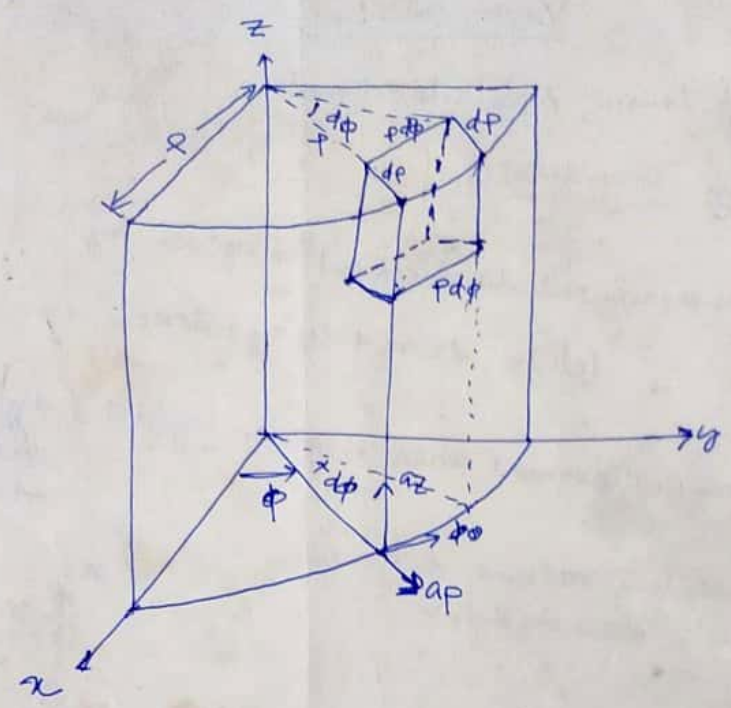

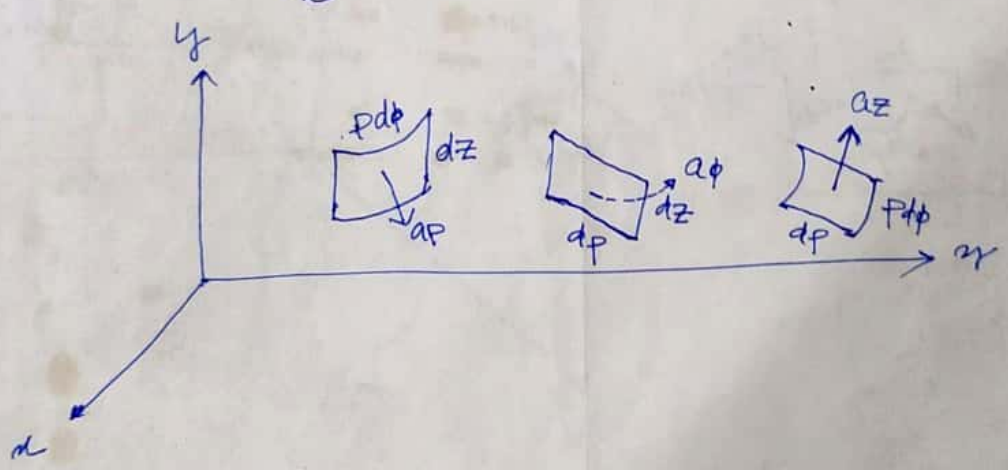

Spherical Co-ordinate :

The differential displacement is

$$(dL) = dr\,a_r + r\,d\theta\,a_\theta + r\sin\theta\,d\phi\,a_\phi$$

The differential normal area is

$$ds = r^2 \sin\theta\, d\theta\, d\phi\, a_r$$
$$r\sin\theta\, dr\, d\phi\, a_\theta$$
$$r\, dr\, d\theta\, a_\phi$$

The differential volume is $dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$.

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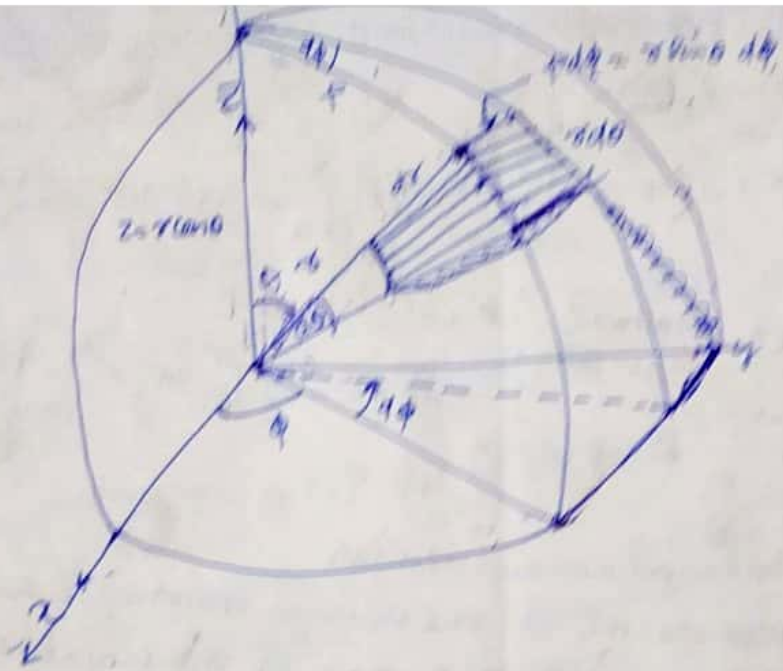

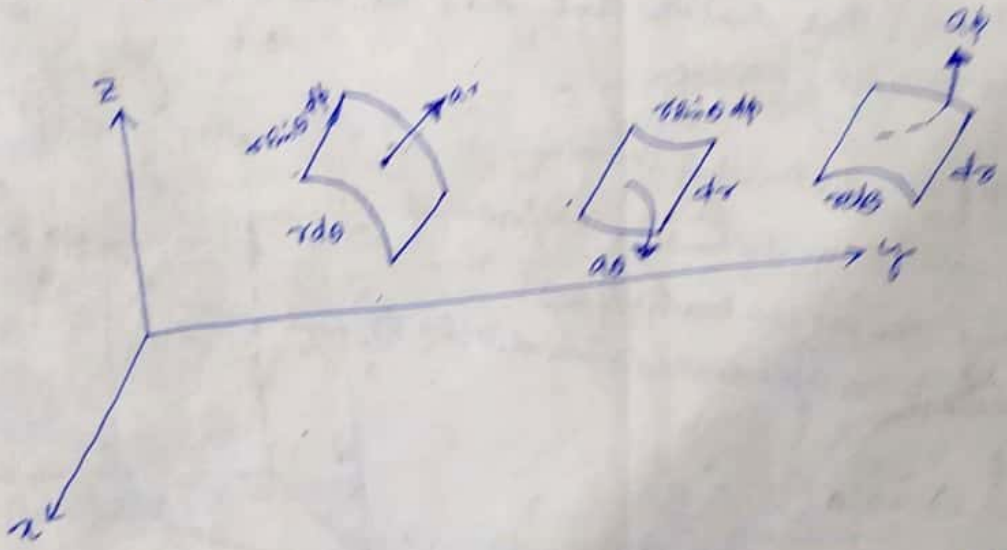

8. (1) Obtain the expression for volume of the sphere of radius a from the differential volume.

Sol^n - $dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$.

$$V = \iiint_{r=0 \, \theta=0 \, \phi=0}^{a \, \pi \, 2\pi} r^2 \sin\theta \, dr \, d\theta \, d\phi = \frac{4}{3}\pi a^3.$$

(2) Use cylindrical co-ordinates to find the area of the curved surface of a right circular cylinder of radius 'a' and height 'h'. Obtain the volume.

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Volume integral:

The volume integral is defined as

$$\int_V \rho_v \, dv$$

as the volume integral of the scalar ρ_v over the volume V.

DEL Operator:

The del operator ∇ is defined as the vector differential operator.

In cartesian co-ordinates.

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

This vector differential operator is known as the gradient operator.

1. The gradient of a scalar v (∇v)
2. The divergence of a vector ($\nabla \cdot v$)
3. The curl of a vector ($\nabla \times v$)
4. The laplacian of a scalar v ($\nabla^2 v$)

∇ can be expressed in terms of cylindrical co-ordinate and spherical co-ordinate

$$\nabla = \frac{\partial}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} a_\phi + \frac{\partial}{\partial z} a_z \rightarrow \text{cylindrical co-ordinate}$$

$$\nabla = \frac{\partial}{\partial r} a_r + \frac{1}{r} \frac{\partial}{\partial \theta} a_\theta + \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} a_\phi$$

(Spherical-Co-ordinate)

$$\rho = \sqrt{x^2+y^2}, \quad \tan\phi = y/x, \quad z = z$$

$$r = \sqrt{x^2+y^2+z^2}, \quad \theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z}, \quad \phi = \tan^{-1}(y/x)$$

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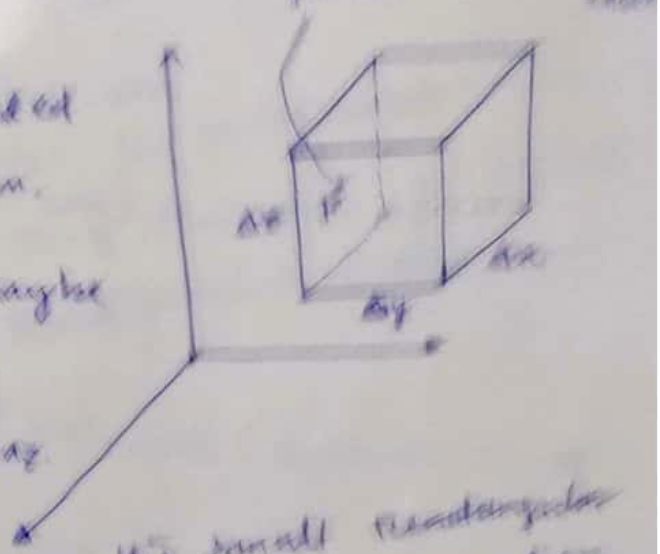
The divergence of A at a given point P is the outward flux per unit volume as the volume shrinks about P

$$\text{Hence } \operatorname{div} A = \nabla \cdot A = \lim_{\nabla v \to 0} \frac{\oint A \cdot ds}{\nabla v}$$

where ∇v is the volume enclosed by the closed surface S in which P is located



Let us consider any point P located by a cartesian co-ordinate system.

The value of A at the point P may be expressed in cartesian co-ordinates

$$A = A_0 = A_{x0} a_x + A_{y0} a_y + A_{z0} a_z$$

Let us assume our closed surface the small rectangular box, centered at P having sides of length Δx, Δy and Δz

Now, we can find out the integral over the closed surface the integral must be broken up into six integrals, one over each face

$$\oint A \cdot ds = \left[\int_{front} + \int_{back} + \int_{left} + \int_{right} + \int_{top} + \int_{bottom} \right] A \cdot ds$$

Since surface is very small, so A is ~~essentially~~ const

$$\int_{front} A \cdot ds = A_{front} \cdot \Delta S_{front} \qquad \Delta S_{front} = \Delta y \, \Delta z \, a_x$$

$$= A_{x\,front} \, \Delta y \, \Delta z$$

2-dimensional Taylor series expansion of A_x about P.

We have only to approximate the value of A_x at the front face. The front face is at a distance of $\frac{\Delta x}{2}$ from P and hence ... Δx ... at change of A_x within

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$$= A_{x0} + \frac{\Delta x}{2} \times \frac{\partial A_x}{\partial x}\Big|_P \quad \text{——— (1)}$$

$$\int_{\text{front}} A \cdot ds = \left(A_{x0} + \frac{\Delta x}{2} \frac{\partial A_x}{\partial x}\Big|_P\right) \Delta y \Delta z$$

Now consider the integral over the back surface.

$$\int_{\text{back}} A \cdot ds = A_{\text{back}} \cdot \Delta s_{\text{back}} \qquad \Delta s_{\text{back}} = \Delta y \Delta z (-a_x)$$

$$= -A_{x,\text{back}} \Delta y \Delta z$$

$$A_{x,\text{back}} = A_{x0} - \frac{\Delta x}{2} \times \frac{\partial A_x}{\partial x}\Big|_P$$

$$\int_{\text{back}} A \cdot ds = -\left(A_{x0} - \frac{\Delta x}{2} \frac{\partial A_x}{\partial x}\Big|_P\right) \Delta y \Delta z$$

$$= \left(-A_{x0} + \frac{\Delta x}{2} \frac{\partial A_x}{\partial x}\Big|_P\right) \Delta y \Delta z \quad \text{——— (2)}$$

If we combine these two integral, we have

$$\int_{\text{front}} A \cdot ds + \int_{\text{back}} A \cdot ds = \frac{\partial A_x}{\partial x}\Big|_P \Delta x \Delta y \Delta z \quad \text{——— (3)}$$

By exactly the same process, we can found out

$$\int_{\text{right}} A \cdot ds + \int_{\text{left}} A \cdot ds = \frac{\partial A_y}{\partial y}\Big|_P \Delta x \Delta y \Delta z \quad \text{——— (4)}$$

$$\text{and} \quad \int_{\text{top}} A \cdot ds + \int_{\text{bottom}} A \cdot ds = \frac{\partial A_z}{\partial z}\Big|_P \Delta x \Delta y \Delta z \quad \text{——— (5)}$$

$$\oint A \cdot ds = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)\Big|_P \Delta x \Delta y \Delta z$$

$$= \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)\Big|_P \cdot \Delta v \qquad \Delta v = \Delta x \Delta y \Delta z$$

$$\lim_{\Delta v \to 0} \frac{\oint A \cdot ds}{\Delta v} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\Rightarrow \boxed{\text{div} A = \nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

Similarly cylindrical coordinate

$$\boxed{\nabla \cdot A = \frac{1}{\rho} \frac{\partial}{\partial \rho}[\rho A_\rho] + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}}$$

spherical coordinate

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Properties of divergence of a vector field.

① It produces a scalar field.

② The divergence of a scalar V, makes no sense.

③ $\nabla\cdot(A+B) = \nabla\cdot A + \nabla\cdot B$

④ $\nabla\cdot(VA) = V\,\nabla\cdot A + A\cdot\nabla V$

physical significance of divergence -

Divergence Theorem

The divergence theorem states that the total outward flux of a vector field A through the closed surface S is equal to the volume integral of the divergence of A.

ie $$\boxed{\oint_S A\cdot ds = \int_V \nabla\cdot A\, dV}$$

Proof:

RHS. $\nabla\cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ where $dV = dx\,dy\,dz$

$$\int_V (\nabla\cdot A)\,dV = \iiint\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)dx\,dy\,dz$$

$$= \iiint \frac{\partial A_x}{\partial x}\,dx\,dy\,dz + \iiint \frac{\partial A_y}{\partial y}\,dx\,dy\,dz + \iiint \frac{\partial A_z}{\partial z}$$

But $\int \frac{\partial A_x}{\partial x}dx = A_x$, $\int \frac{\partial A_y}{\partial y}dy = A_y$, $\int \frac{\partial A_z}{\partial z}dz = A_z$.

$$= \iint A_x\,dy\,dz + \iint A_y\,dx\,dz + \iint A_z\,dx\,dy$$

$ds_x = dy\,dz$ = Surface area of yz plane with unit vector a_x

$ds_y = dx\,dz$ = surface area of xy plane with unit vector a_y

ds_z = surface area of xz plane

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$$\int_V (\nabla \cdot A)\, dV = \iint [A_x\, ds_x + A_y\, ds_y + A_z\, ds_z]$$

$$= \iint A \cdot ds$$

$$\Rightarrow \boxed{\int_V (\nabla \cdot A)\, dV = \int_S A \cdot ds}$$

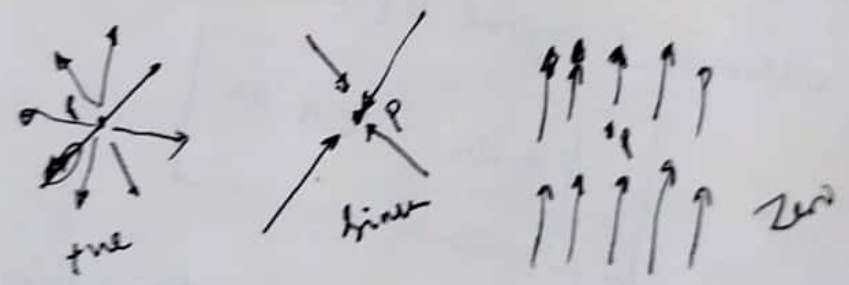
Alternative method

Physical Significance

① Positive divergence → act as source → vector diverges (as spread out)

② Negative divergence → sink (convergence)

③ Zero divergence → neither source nor sink



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static (or time-invariant) electric
An electrostatic field is produced by a static charge distribution. Ex- field is found in cathode ray tube.

Coloumb's Law :

The law states that the force F betn two point charges Q_1 and Q_2 is

1. directly proportional to the product Q_1Q_2 of the charges
2. Inversely proportional to the square of the distance (R) betn them.
3. directed along the line joining the charges.

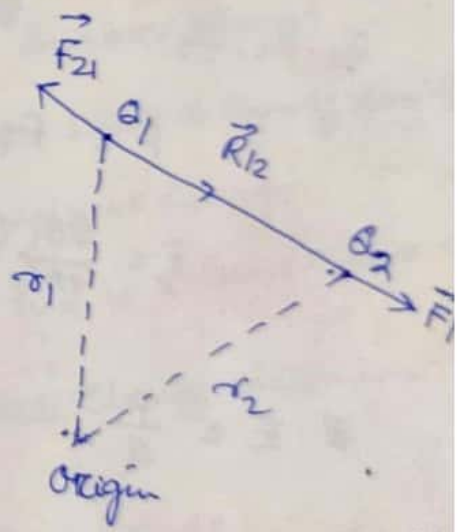
$$F \propto \frac{Q_1Q_2}{R^2} \qquad F = K\frac{Q_1Q_2}{R^2} \text{ where } K = \frac{1}{4\pi\epsilon_0} = 9\times10^9 \, m/F.$$

$$\epsilon_0 = 8.85\times10^{-12} = \frac{10^{-9}}{36\pi} \, F/m.$$

$$\boxed{F = \frac{Q_1Q_2}{4\pi\epsilon_0 R^2}}$$

If point charges Q_1 and Q_2 are located at points having position vector r_1 and r_2 then $\vec{F} = \frac{Q_1Q_2 \vec{R}_{12}}{4\pi\epsilon_0 |R_{12}|^2}$ where $\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$

Then the force $\vec{F}_{12}$ on Q_2 due to Q_1 is given by



$$F_{12} = \frac{Q_1Q_2}{4\pi\epsilon_0} \cdot \frac{\vec{R}_{12}}{|\vec{R}_{12}|^3}$$

$$\boxed{\vec{F}_{12} = \frac{Q_1Q_2}{4\pi\epsilon_0} \cdot \frac{\vec{r}_2 - \vec{r}_1}{|r_2 - r_1|^3}}$$

But $\vec{F}_{12} = -\vec{F}_{21}$ as $q_{12} = -q_{21}$.

If both the charges are of opposite polarity then the force acting on them is attractive where as both charges are same sign the force is repulsive force.

A point charge mean a charge that is located on a body whose are much smaller than other relevant dimens

* Two charged bodies that are very small in size compared to their separation so that they can be considered as point ch

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Electric Field intensity:

The electric field intensity (E) is the force per unit charge when placed in the electric field.

Then $\vec{E} = \lim_{Q \to 0} \frac{\vec{F}}{|Q|}$

$$\vec{E} = \frac{\vec{F}}{|Q|}.$$

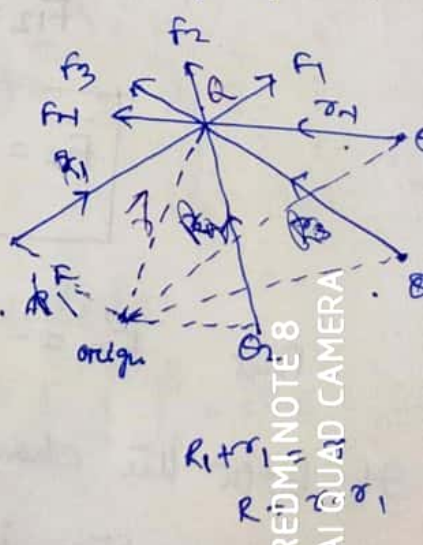
The electric field intensity 'E' obviously in the dir^n of force (F).

If the system has more than 2 charges then we can use the principle of superposition to determine the force on a particular charges.

The principle states that if there are N charges $Q_1, Q_2, Q_3 \ldots Q_N$ located respectively at points with position vectors $r_1, r_2 \ldots r_N$, the resultant force F on a charge Q located at point r is the vector sum of the force exerted on Q by other charges $Q_1, Q_2, Q_3 \ldots Q_N$.

$$F = f_1 + f_2 + \ldots + f_N.$$

$$F = \frac{QQ_1(r-r_1)}{4\pi\epsilon_0 |r-r_1|^3} + \frac{QQ_2(r-r_2)}{4\pi\epsilon_0 |r-r_2|^3} + \ldots + \frac{QQ_N(r-r_k)}{4\pi\epsilon_0 |r-r_k|^3}.$$



$$E = \frac{F}{Q}.$$

$$E = \frac{Q_1(r-r_1)}{4\pi\epsilon_0 |r-r_1|^3} + \frac{Q_2(r-r_2)}{4\pi\epsilon_0 |r-r_2|^3} + \ldots + \frac{Q_N(r-r_k)}{4\pi\epsilon_0 |r-r_k|^3}.$$

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$$\boxed{E = \frac{Q}{4\pi\epsilon_0}\frac{(r-r')}{|r-r'|^3}}$$

For N point charges $Q_1, Q_2, Q_3 \ldots\ldots Q_N$ located at $r_1, r_2, r_3 \ldots r_N$ the electric field intensity at point r is obtained

$$E = \frac{Q_1(r-r_1)}{4\pi\epsilon_0|r-r_1|^3} + \frac{Q_2(r-r_2)}{4\pi\epsilon_0|r-r_2|^3} + \ldots\ldots + \frac{Q_N(r-r_N)}{4\pi\epsilon_0|r-r_N|}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_{K=1}^{N}\frac{Q_K(r-r_K)}{|r-r_K|^3}}$$

<u>Electric Field due to Contineous Charage Distribution.</u>

If the charge element dQ and the total charge Q due to these charge distribution then

$$dQ = \rho_L dl.$$

$$Q = \int_L \rho_L dl \quad \text{where } \rho_L \text{ is line charge density.}$$

$$\rho_s = \frac{dQ}{ds} \qquad Q = \int_s \rho_s ds = \text{Surface Charge}$$

ρ_s = Surface charge density.

$$\rho_v = \frac{dQ}{dv} \qquad Q = \int_v \rho_v dv = \text{volume Charge}$$

ρ_v = is volume charge density

The electric field intensity due to each charge distributions ρ_L, ρ_s & ρ_v are

$$E = \int \frac{\rho_L dl}{4\pi\epsilon_0 R^2} a_R \quad \text{(Line charge)} \qquad E = \int \frac{\rho_s ds}{4\pi\epsilon_0 R^2} a_R \text{ (Surface charge)}$$

$$E = \int \frac{\rho_v dv}{4\pi\epsilon_0 R^2} a_R \rightarrow \text{volume charge.}$$

Line charge for which charge is distributed along a Line (Straight line curved) ρ_L (C/m).

Surface charge distribution for which charge is distributed on surface (Planner or Co planner) ρ_s C/m²

↳ Surface charge

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Electric Field due to Line Charge:

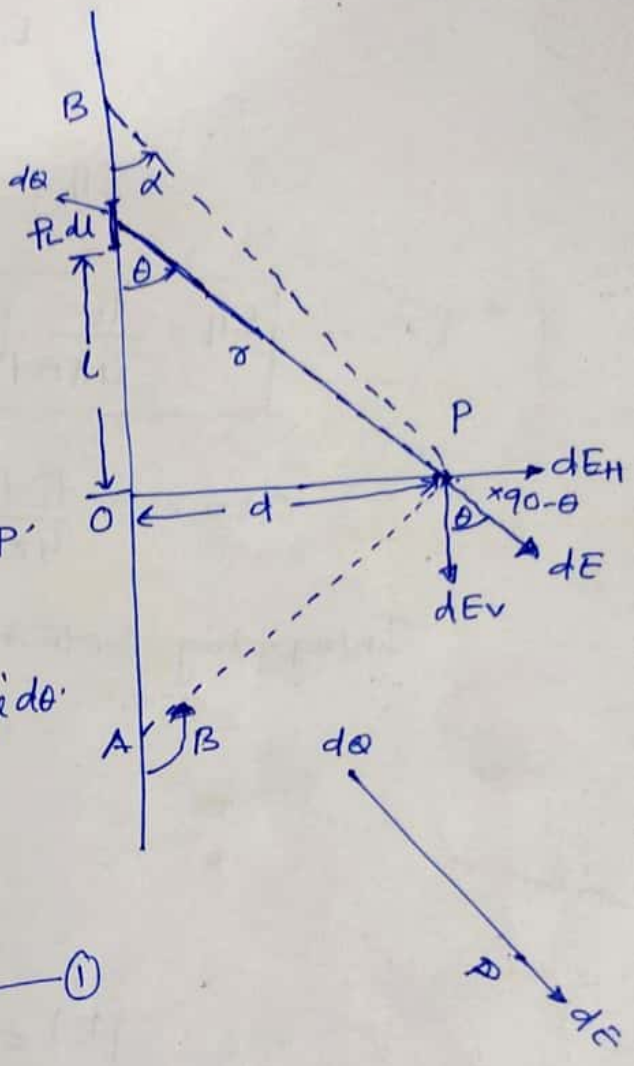
Let us consider a line charge with a uniform charge density ρ_L extending from A to B. Consider a point charge dQ at a distance 'l' from the origin.

So $dQ = \rho_L dl$.

'O' is not a mid point of AB.

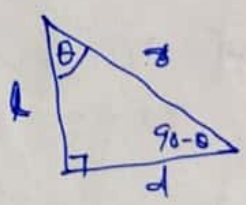
dE be the electric field at point 'P' due to charge 'dQ'.

'r' is the distance bet^n the charge dQ and the point 'P'.



$$\text{So } dE = \frac{dQ}{4\pi\epsilon_0 r^2} = \frac{\rho_L dl}{4\pi\epsilon_0 r^2} \quad \text{—— (1)}$$

From this triangle $\tan\theta = \frac{d}{l}$



$$l = \frac{d}{\tan\theta} = d\cot\theta.$$

$$dl = -d\,\mathrm{cosec}^2\theta\, d\theta \quad \text{—— (2)}$$

$$\sin\theta = \frac{d}{r} \Rightarrow r = d\,\mathrm{cosec}\,\theta \quad \text{—— (3)}$$

$$dE = \frac{\rho_L(-d\,\mathrm{cosec}^2\theta\, d\theta)}{4\pi\epsilon_0 (d\,\mathrm{cosec}\,\theta)^2} = \frac{-\rho_L d\theta}{4\pi\epsilon_0 d}$$

From the fig. $dE_H = dE\cos(90-\theta) = dE\sin\theta$

$dE_V = dE\cos\theta.$

i.e. $\rho_L d\theta$ or θ

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Integrating both side,

$$E_H = -\frac{\rho_L}{4\pi\epsilon_0 d}\int_{\beta}^{\alpha} \sin\theta \, d\theta$$

$$E_H = -\frac{\rho_L}{4\pi\epsilon_0 d}\left[-\cos\theta\right]_{\beta}^{\alpha} = \frac{\rho_L}{4\pi\epsilon_0 d}\left[\cos\alpha - \cos\beta\right]$$

$$\boxed{E_H = \frac{\rho_L}{4\pi\epsilon_0 d}\left[\cos\alpha - \cos\beta\right]} \quad \text{--- (5)}$$

$$dE_V = -\frac{\rho_L d\theta}{4\pi\epsilon_0 d}\cos\theta = -\frac{\rho_L \cos\theta}{4\pi\epsilon_0 d}d\theta.$$

Integrating both sides, $E_V = -\frac{\rho_L}{4\pi\epsilon_0 d}\int_{\beta}^{\alpha}\cos\theta \, d\theta$

$$= -\frac{\rho_L}{4\pi\epsilon_0 d}\left[\sin\theta\right]_{\beta}^{\alpha}$$

$$\boxed{E_V = \frac{\rho_L}{4\pi\epsilon_0 d}\left[\sin\beta - \sin\alpha\right]} \quad \text{--- (6)}$$

$$|E| = \sqrt{(E_H)^2 + (E_V)^2}.$$

If 'O' is the mid point $\alpha + \beta = \pi$, $\alpha = \pi - \beta$.

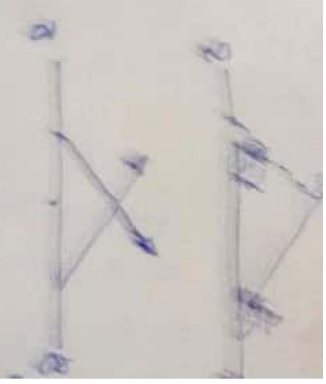
$$E_V = \frac{\rho_L}{4\pi\epsilon_0 d}\left[\sin\beta - \sin(\pi - \beta)\right] = 0.$$

$$|E| = \sqrt{E_H^2 + E_V^2} = E_H.$$

Therefore resultant electric field is normal to the line charge. As length $\to \infty$, $\alpha = 0$, $\beta = \pi$.

So $E_H = \frac{\rho_L}{4\pi\epsilon_0 d}\left[1 - (-1)\right] = \frac{\rho_L}{2\pi\epsilon_0 d}$

$$\boxed{E = \frac{\rho_L}{2\pi\epsilon_0 d}}$$



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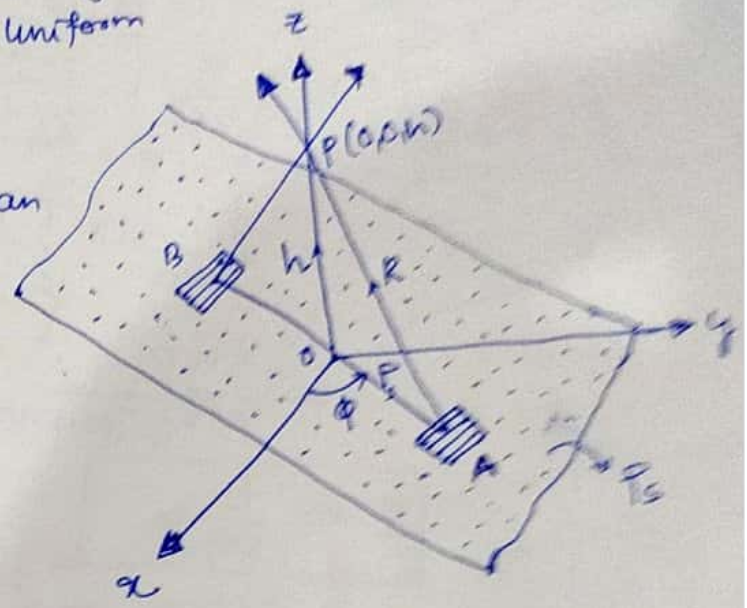
Electric Field due to Surface Charge.

Let us consider an infinite sheet of charge in the xy plane with uniform charge density ρ_s.



The charge associated with an elemental area ds is given by $dQ = \rho_s ds$.

The contribution to the E field at point P(0,0,h) by the elemental surface

$$dE = \frac{dQ}{4\pi\epsilon_0 R^2} a_R.$$

$$\vec{R} = \rho(-a_\rho) + h a_z.$$

$$|\vec{R}| = \sqrt{\rho^2 + h^2} \qquad a_R = \frac{\vec{R}}{R}.$$

a_z is $\perp$ to the area

$$dQ = \rho_s ds = \rho_s \rho d\phi d\rho.$$

$$dE = \frac{\rho_s \rho d\phi d\rho \left[-\rho a_\rho + h a_z \right]}{4\pi\epsilon_0 \left[\rho^2 + h^2 \right]^{3/2}}.$$

$\vec{E}$ at P due to point charges of the two elements situated diametrically opposite points A and B on the sheet. The two segments A and B have equal in magnitude but the dir^n will be such that the resultant $\vec{E}$ will be directed along OP and away from the sheet charge. Therefore the resultant $\vec{E}$ at P will also be independent of the position of

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So the contribution to E_ρ added up to zero. so that E has only z-component.

$$E = \int dE_z = \frac{\rho_s}{4\pi\epsilon_0}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\infty}\frac{h\rho\, d\rho\, d\phi}{(\rho^2+h^2)^{3/2}}a_z$$

$$= \frac{\rho_s h}{4\pi\epsilon_0}\times 2\pi\int_0^{\infty}\frac{\rho\, d\rho}{[\rho^2+h^2]^{3/2}}a_z$$

$$= -\frac{\rho_s h}{2\epsilon_0}\left[\frac{1}{[\infty+h^2]^{1/2}}-\frac{1}{h}\right]a_z$$

$$\boxed{E = \frac{\rho_s}{2\epsilon_0}a_z}$$

$$\int\frac{\rho\, d\rho}{[\rho^2+h^2]^{3/2}} \qquad t = \rho^2+h^2, \quad dt = 2\rho\, d\rho, \quad \rho\, d\rho = \frac{dt}{2}$$

$$= \frac{1}{2}\int\frac{dt}{t^{3/2}}$$

$$= \frac{1}{2}\cdot\frac{t^{-3/2+1}}{-3/2+1} = -t^{-1/2}$$

$$= -\frac{1}{[\rho^2+h^2]^{1/2}}$$

The above equ^n it is clear that E has only z-component if the charge is in the xy plane.

In general for an infinite sheet of charge

$$\boxed{E = \frac{\rho_s}{2\epsilon_0}a_n}$$

In Parallel plate capacitor, the electric field existing bet^n the two plates having equal and opposite charges is given by

$$E = \frac{\rho_s}{2\epsilon_0}a_n + \frac{-\rho_s}{2\epsilon_0}(-a_n) = \frac{\rho_s}{\epsilon_0}a_n.$$

$$\boxed{E = \frac{\rho_s}{\epsilon_0}a_n}$$

Electric Field due to volume charge:

Let us consider that the volume charge distribution with a uniform charge density ρ_v.

[illegible] with elemental volume dv is

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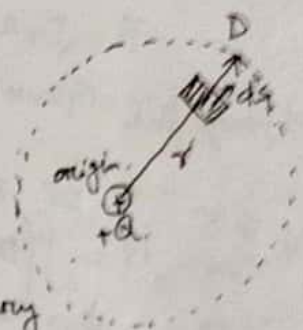
Gauss's law:

Gauss's law state that the total electric flux (ψ) through any closed surface is equal to the total charge enclosed by that surface.

$$\boxed{\psi = Q}$$

Gauss's law is very useful to find out electric field intensity, for this we can construct an imaginary surface called "Gaussian surface". The electric field must be uniform at every point on the surface. The electric field must be normal to the surface is considered.



Let us consider a point charge 'Q' kept at the origin as shown in fig.

Let us consider an infinite elementary area 'ds' at a distance 'r' from the origin.

The displacement density vector and area vector are normal to the surface 'ds'.

Let the flux crossing through the surface ds is $d\psi$.

$$\vec{D} = \frac{d\psi}{ds} \qquad d\psi = \vec{D}\cdot ds$$

$$\psi = \int_s \vec{D}\cdot ds$$

$$\psi = Q = \oint_s D\cdot ds = \int_v \rho_v \, dv$$

$$\Rightarrow \int_v (\nabla\cdot D)\, dv = \int_v \rho_v \, dv$$

$$\boxed{\nabla\cdot D = \rho_v} \quad \text{——— 1- Maxwell's Eqn}$$

The 1st Maxwell's Eqn state the volume charge density is same as

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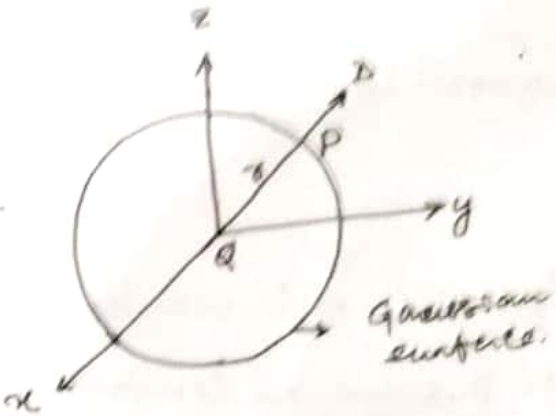
Application of Gauss's law:

(A) A point charge:

Suppose a point charge Q is located at the origin. we can choose a spherical surface containing P. to satisfy symmetry condⁿ.



Since D is everywhere normal to the Gaussian surface

$$\vec{D} = D_r a_r.$$

Applying the Gauss's law.

$$Q = \oint D \cdot ds \qquad ds_r = r^2 \sin\theta \, d\theta \, d\phi \, a_r$$

$$Q = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} D_r \, r^2 \sin\theta \, d\theta \, d\phi = D_r \cdot 4\pi r^2.$$

$$\boxed{D_r = \frac{Q}{4\pi r^2}}$$

$$\boxed{\vec{D} = D_r a_r = \frac{Q}{4\pi r^2} a_r.}$$

$$D = \epsilon_0 E \Rightarrow \boxed{E = \frac{D}{\epsilon_0} = \frac{Q}{4\pi\epsilon_0 r^2} a_r}$$

(B) Infinite line charge:

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$$Q = D_\rho \, 2\pi\rho l$$

$$\boxed{D_\rho = \frac{Q}{2\pi\rho l}}$$

where $Q = \rho_L l$ $\quad D_\rho = \frac{\rho_L l}{2\pi\rho l}$

$$\boxed{D_\rho = \frac{\rho_L}{2\pi\rho}}$$

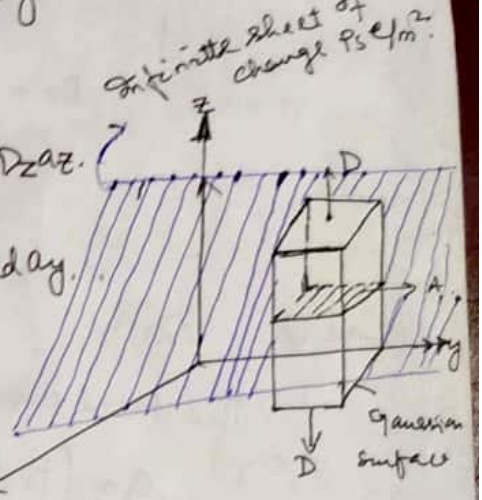
$$D = \epsilon_0 E \Rightarrow D_\rho = \epsilon_0 E_\rho$$

$$E_\rho = \frac{D_\rho}{\epsilon_0} \qquad \boxed{E_\rho = \frac{\rho_L}{2\pi\epsilon_0\rho}}$$

Thus in general $\boxed{\vec{D} = \frac{\rho_L}{2\pi\rho} a_\rho}$

Infinite sheet of charge:

Consider the infinite sheet of uniform charge ρ_s C/m² lying on the $z=0$ plane (xy-plane). To determine 'D' at point P, we choose a rectangular box that is cut symmetrically by the sheet of charge.



As $\vec{D}$ is normal to the sheet $\vec{D} = D_z a_z$.

$\vec{D}$ has no component along a_x and a_y.

Applying Gauss's law,

$$\psi = \oint D \cdot ds$$

$$= D_z \left[\int_{top} ds + \int_{bottom} ds \right]$$

If the top and bottom area of each box is A.

$$\psi = D_z [A + A]$$

$$\psi = D_z \cdot 2A$$

$$\psi = Q \qquad Q = D_z 2A \qquad D_z = \frac{Q}{2A}$$

But $Q = \int \rho_s ds = \rho_s A$

$$D_z = \frac{\rho_s A}{2A} = \frac{\rho_s}{2}. \qquad \vec{D} = D_z a_z = \frac{\rho_s}{2} a_z$$

$$\boxed{\vec{D} = \frac{\rho_s}{2} a_z} \qquad \boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0} a_z}$$

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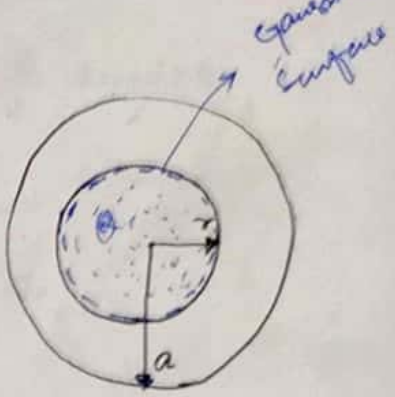
Consider a sphere of radius 'a' with a uniform charge ρ_v C/m³. Now to determine D everywhere we construct Gaussian surface for cases $r \le a$ and $r \geq a$ separately. Since the charge has spherical symmetry.



Case ①. $r \le a$.

The total charge enclosed by the spherical surface of radius 'r' is

$$Q = \int \rho_v \, dv = \rho_v \int_0^{2\pi}\int_0^{\pi}\int_{r=0}^{r} r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= \rho_v \cdot \frac{4}{3}\pi r^3.$$

and $\psi = \oint D \cdot ds = \oint D_r a_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, a_r = D_r \cdot 4\pi r^2.$

But $\psi = Q$.

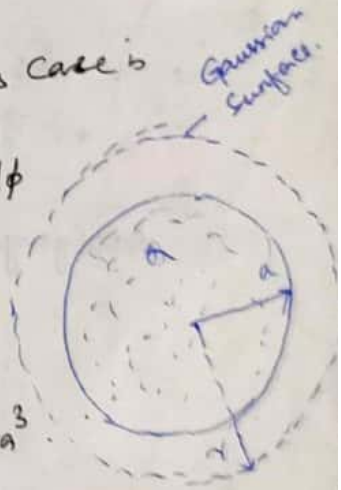
$$D_r \cdot 4\pi r^2 = \rho_v \frac{4}{3}\pi r^3$$

$$D_r = \frac{r}{3}\rho_v. \qquad \vec{D} = D_r a_r$$

$$\boxed{\vec{D} = \frac{r}{3}\rho_v a_r} \quad 0 \le r \le a.$$

Case ②. $r \geq a$.

The total charge enclosed by the surface in this case is



$$Q = \int \rho_v \, dv = \rho_v \int dv = \rho_v \int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$Q = \rho_v \cdot \frac{4}{3}\pi a^3.$$

But $\psi = \oint D \cdot ds = D_r \cdot 4\pi r^2.$

$$\psi = Q \Rightarrow D_r 4\pi r^2 = \rho_v \frac{4}{3}\pi a^3$$

$$D_r = \frac{a^3}{3r^2}\rho_v.$$

$$\vec{D} = D_r a_r \Rightarrow \boxed{\vec{D} = \frac{a^3}{3r^2}\rho_v a_r}$$

So D. everywhere is given by

$$D = \begin{cases} \frac{r}{3}\rho_v a_r & 0 \le r \le a \\ \frac{a^3}{3r^2}\rho_v a_r & r \geq a. \end{cases}$$

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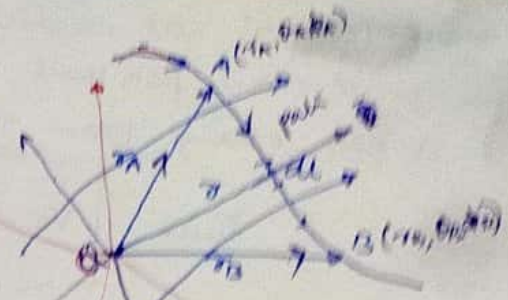
Electric Potential:

Suppose when a point charge Q move from point A to B in an electric field E, then the force on Q is $\vec{F} = Q\vec{E}$



so the work is done in displacing the charge by dl is $dW = -F \cdot dl$

$= -Q\vec{E} \cdot dl$ (-ve indicates work is done by an external agent)

So the total work done is obtain in moving Q from A to B is

$$W = -Q \int_A^B E \cdot dl$$

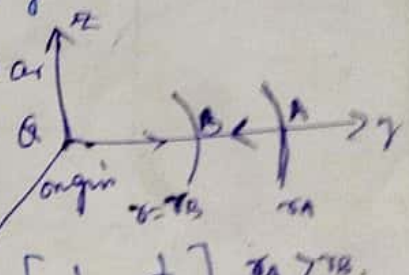
$$\frac{W}{Q} = -\int_A^B E \cdot dl$$

$$V_{AB} = -\int_A^B E \cdot dl$$ = potential difference betn points A and B

Potential at any point due to a single point charge Q.

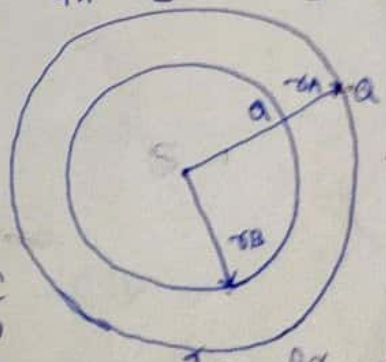
If the electric field E due to a point charge 'Q' located at the origin, then $\vec{E} = E_r a_r$, $dl = dr\, a_r$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} a_r.$$



$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2} a_r \cdot dr\, a_r = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad r_A > r_B.$$

$$\boxed{V_{AB} = V_B - V_A}$$



Where V_B and V_A are the potentials at B and A.

we assume the potential at infinity is zero → initial point 'A' taken at infinity

Thus if $V_A = 0$ as $r \to \infty$. $V = -\int_\infty^r E\,dl = -\int_\infty^r \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_\infty^r$ (work is done by an external agent $r_A - r_B$.)

The potential at any point ($r_B \to r$) due to a point charge 'Q' located at the origin is $\boxed{V = \frac{Q}{4\pi\epsilon_0 r}}$ $= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$

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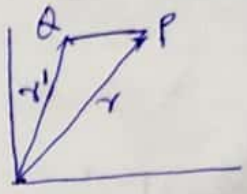
The potential at a distance r from the point charge is the work done per unit charge by an external agent in transfer a test charge from infinity to that point.

Thus $$\boxed{V = -\int_{\infty}^{r} E \cdot dl}$$

(b) Potential due to various type of charge distribution.

If the point charge Q is not located at the origin, but at a point whose position vector is r', then.



$$\boxed{V(r) = \frac{Q}{4\pi\epsilon_0 |r - r'|}}$$

If there are n point charges $Q_1, Q_2, Q_3 \ldots Q_n$ located at points with position vectors $r_1, r_2, r_3 \ldots r_n$, then we can apply superposition principle then the resultant potential at r is

$$V(r) = \frac{Q_1}{4\pi\epsilon_0 |r - r_1|} + \frac{Q_2}{4\pi\epsilon_0 |r - r_2|} + \ldots + \frac{Q_N}{4\pi\epsilon_0 |r - r_n|}$$

$$\boxed{V(r) = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^{n} \frac{Q_k}{|r - r_k|}}$$

For continuous charge distribution, then

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_L \frac{\rho_L(r')\, dl'}{|r - r'|} \quad \text{(Line charge)}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_S \frac{\rho_s(r')\, ds'}{|r - r'|} \quad \text{(Surface charge)}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho_v(r')\, dv'}{|r - r'|} \quad \text{(volume charge)}$$

Relationship betn E & V.

Since $V = -\int E \cdot dl$

$dV = -\vec{E} \cdot \vec{dl}$

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But $dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$ ——— ②

Comparing eqn ① & ②.

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

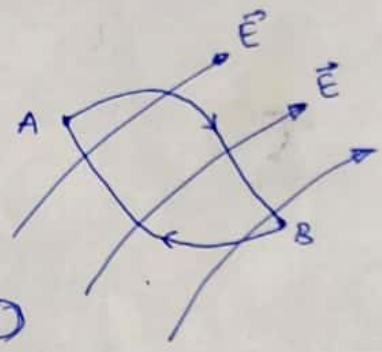
$$\vec{E} = E_x a_x + E_y a_y + E_z a_z$$
$$= -\frac{\partial V}{\partial x}a_x - \frac{\partial V}{\partial y}a_y - \frac{\partial V}{\partial z}a_z$$

$$\boxed{\vec{E} = -\nabla V}$$

The electric field intensity is the gradient of V. The negative sign indicates that the dir^n of E is opposite to that in V increase.

Conservative nature of electrostatic field.

Since we know that the potential difference betn points A and B is independent of the path taken. Hence $V_{AB} = -V_{BA}$.



$$V_{AB} + V_{BA} = 0.$$

$$\Rightarrow \boxed{\oint E \cdot dl = 0} \quad \text{——— ①}$$

This shows that the line integral of E along a closed path as shown in fig must be zero.

That means physically which implies that no work is done in moving a charge along a closed path in an electrostatic field.

Applying Stoke's theorem.

$$\oint E \cdot dl = \int (\nabla \times E) \cdot ds = 0$$

$$\boxed{\nabla \times E = 0} \quad \text{——— ②}$$

So any vector field that satisfied eqn^n ① & (2) is said to be conservative or irrotational.

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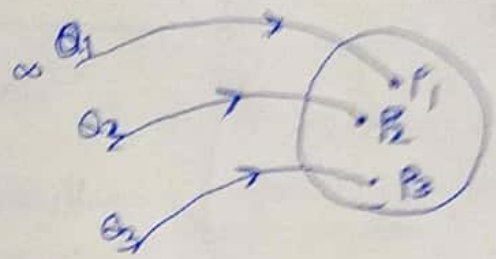
Energy density in Electrostatic Fields:

To determine the energy present in an assembly of charges, we should first determine the amount of work done by an external source to assemble them.

Let us consider 3 charges Q_1, Q_2, Q_3 are placed in an empty space. No work is required to transfer Q_1 from infinity to P_1 as the space is initially charge free and there is no electric field.



The work is done in transfering Q_2 from infinity to P_2 is equal to the product of the charges Q_2 and potential V_{21} at P_2 due to Q_1 $W_2 = Q_2 V_{21}$

* $W = -Q\int E\,dl$
$= Q\int -E\,dl$
$= QV$

Similarly the work done transfering Q_3 at P_3 is equal to $Q_3(V_{32} + V_{31})$. where V_{32} and V_{31} are the potential at P_3 due to Q_2 and Q_1 respectively.

* In V_{21}, 2 subscript indicate the location, 1 subscript indicates the source.
V_{21} = Potential at location of Q_2 due to Q_1

Hence the total work done in transfering the three charges $W_E = W_1 + W_2 + W_3$

$$= 0 + Q_2 V_{21} + Q_3(V_{31} + V_{32}) \quad \text{——— (1)}$$

when the charges position are reverse then

$$W_2 = Q_2 V_{21} = Q_2 \cdot \frac{Q_1}{4\pi\epsilon_0 R_{12}} = Q_1 \frac{Q_2}{4\pi\epsilon_0 R_{21}} = Q_1 V_{12}$$

where R_{12} and R_{21} each represent the scalar distance bet^n Q_1 and Q_2 charges.

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$$WE = Q_1 V_{12} + Q_1 V_{13} + Q_2 V_{23}$$

Here charges were positioned in reverse order.

Then adding ① + ②

$$2WE = Q_2 V_{21} + Q_3 (V_{31} + V_{32}) + Q_1 V_{12} + Q_1 V_{13} + Q_2 V_{23}.$$

$$= Q_1 [V_{12} + V_{13}] + Q_2 [V_{21} + V_{23}] + Q_3 [V_{31} + V_{32}]$$

$$= Q_1 V_1 + Q_2 V_2 + Q_3 V_3.$$

$$\boxed{WE = \frac{1}{2} [Q_1 V_1 + Q_2 V_2 + Q_3 V_3]}$$

Where V_1, V_2, V_3 are total potential at P_1, P_2 & P_3 respectively.

$$V_1 = V_{12} + V_{13}$$
$$V_2 = V_{21} + V_{23}$$
$$V_3 = V_{31} + V_{32}.$$

For n number of charge is distributed system, then

$$\boxed{WE = \frac{1}{2} \sum_{K=1}^{n} Q_i V_i}$$

If there is a continuous charge distribution.

$$WE = \frac{1}{2} \int_L \rho_L V dl \text{ (line charge)}$$

$$WE = \frac{1}{2} \int_V \rho_s V ds \text{ (surface charge)}$$

$$= \frac{1}{2} \int_V \rho_v V dv \text{ (volume charges)}$$

$$WE = \frac{1}{2} \int \rho_v V dv = \frac{1}{2} \int (\nabla \cdot D) V dv$$

$$= \frac{1}{2} \int (\nabla \cdot VD) dv - \frac{1}{2} \int_V (D \cdot \nabla V) dv$$

$$= \frac{1}{2} \oint_S (VD) \cdot ds - \frac{1}{2} \int_V (D \cdot \nabla V) dv$$

$\nabla \cdot VA$ — V is a scalar, A is any vector

$= A \cdot \nabla V + V(\nabla \cdot A)$

$(\nabla \cdot A) V = \nabla \cdot VA - A \cdot \nabla V$

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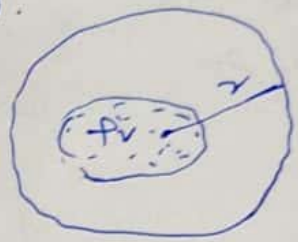
Since we know that V varies as $\frac{1}{r}$ and D as $\frac{1}{r^2}$ for point charges. V varies as $\frac{k}{r^2}$ and D as $\frac{k}{r^3}$ for dipole and so on. Hence VD in the first term on the right hand side in the above equ must vary at least as $\frac{k}{r^3}$ while ds varies as r^2. Consequently the first integral in the above tends to zero as the surface becomes large. Therefore $\lim_{r \to \infty} \oint V D \cdot ds = 0$.



So $$W_E = -\frac{1}{2}\int_V (D \cdot \nabla V)\, dV$$

Since $E = -\nabla V$

$$\boxed{W_E = \frac{1}{2}\int_V (D \cdot E)\, dV}$$

$D = \epsilon_0 E$

Since the enclosing sphere become very large the enclosed volume charge looks like a point charge.

$$W_E = \frac{1}{2}\int \epsilon_0 E^2\, dV$$

$$dW_E = \frac{1}{2} D \cdot E\, dV$$

$$\frac{dW_E}{dV} = \frac{1}{2} D \cdot E$$

$$\boxed{w_e = \frac{1}{2} D \cdot E}$$ = Electrostatic energy density

$$\boxed{w_e = \frac{1}{2} D \cdot E = \frac{1}{2}\epsilon_0 E^2 = \frac{D^2}{2\epsilon_0}}$$

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Poission's and Laplace's equation:-

we know from Gauss's law

$$\nabla \cdot D = \nabla \cdot \epsilon E = \rho_v$$

and $E = -\nabla V$

$$\nabla \cdot \epsilon E = \rho_v$$

$$\Rightarrow \nabla \cdot (-\nabla V) = \frac{\rho_v}{\epsilon} \Rightarrow \boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \rightarrow \text{for an homogenous medium.}$$

This equation is known as Poission's equ.

For a charge free ~~region~~ ~~Prob~~ medium (insulator) $\rho_v = 0$.

$$\boxed{\nabla^2 V = 0}$$

This eqn is known as Laplace's equation.

Thus the Laplace's equation in cartesian, cylindrical or spherical co-ordinates respectively is given by

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 V}{\partial \phi^2} = 0$$

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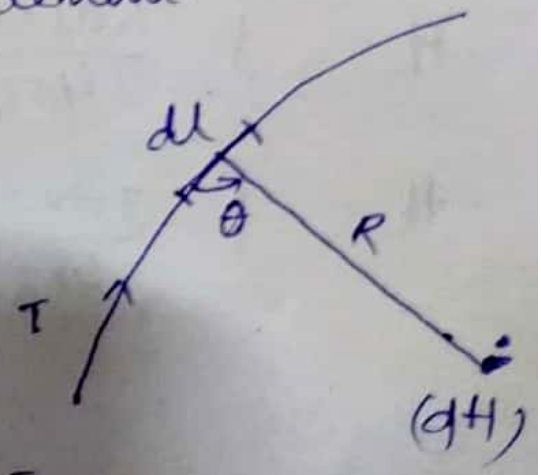
velocity, a [illegible]

These fields are also produced by constant [illegible] direct current. The current flow may be due to magnetization currents as in permanent magnets, or beam currents in vacuum tubes or conduction currents as in current carrying wires.

Biot-Savart's law:

Biot-Savart's law states that ~~the magnetic~~ at any point the magnitude of magnetic field intensity (H) produced by differential element is proportional to product of current and magnitude of differential length (dl) and sine of angle θ betn element and line joining the element and the point 'P'. is inversely proportional to the square of distance 'R' betn 'P' and the element.



$$dH \propto \frac{I\,dl \sin\theta}{R^2}$$

$$dH = K \frac{I\,dl \sin\theta}{R^2}$$

where K is the const of proportionality.

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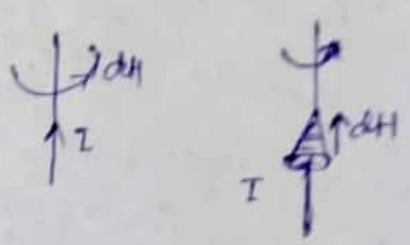
In SI units $K = \frac{1}{4\pi}$

$$dH = \frac{I\,dl \sin\alpha}{4\pi R^2}$$

If we apply vector form then

$$dH = \frac{I\,dl \times a_R}{4\pi R^2} = \frac{I\,dl \times \vec{R}}{4\pi R^3} \qquad \vec{a}_R = \frac{\vec{R}}{|\vec{R}|}$$

The dir^n of dH can be determined by the right hand rule. With the right hand thumb pointing in the dir^n of the current, the right hand fingers encircling the wire in the dir^n of dH.

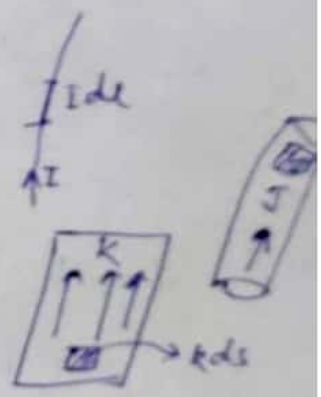


If we can have different current distribution, line current, surface current and volume current.

$Idl \rightarrow$ line current distribution

$K \rightarrow$ surface current density

$J \rightarrow$ volume current density



Thus the source elements are related as

$$Idl \equiv Kds \equiv Jdv.$$

$$H = \int_L \frac{I\,dl \times a_R}{4\pi R^2} \quad \text{(line current)}$$

$$H = \int_S \frac{K\,ds \times a_R}{4\pi R^2} \quad \text{(surface current)}$$

$$H = \int \frac{J\,dv \times a_R}{\quad} \quad \text{(volume current)}$$

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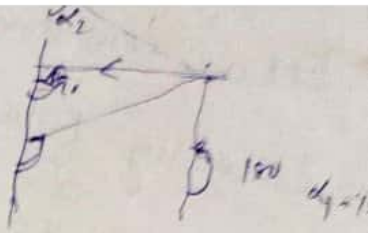
$$\boxed{H = \frac{I}{2\pi\rho}\, a_\phi}$$



Ampere's Circuit law – Maxwell's Equation

Ampere's circuit law states that the line integral of the tangential component of H around a closed path is the same as the net current I_{enc} enclosed by the path.

$$\boxed{\oint H \cdot dl = I_{enc}}$$

In other word circulation of H equals I_{enc}.

Ampere's law is similar to Gauss law and it is easily applied to determine H when the current distribution is symmetrical.

By applying Stoke's theorem, we obtain

$$I_{enc} = \oint_L H \cdot dl = \int_S (\nabla \times H) \cdot ds \quad \text{—— (1)}$$

$$\text{But} \quad I_{enc} = \int_S J \cdot ds \quad \text{—— (2)}$$

Comparing eqn (1) & (2), we get

$$\boxed{\nabla \times H = J} \longrightarrow \text{3rd Maxwell's Eqn}$$

So we can observe that $\nabla \times H = J \neq 0$ i.e magnetostatic field is not conservative.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\ldots\right.$$

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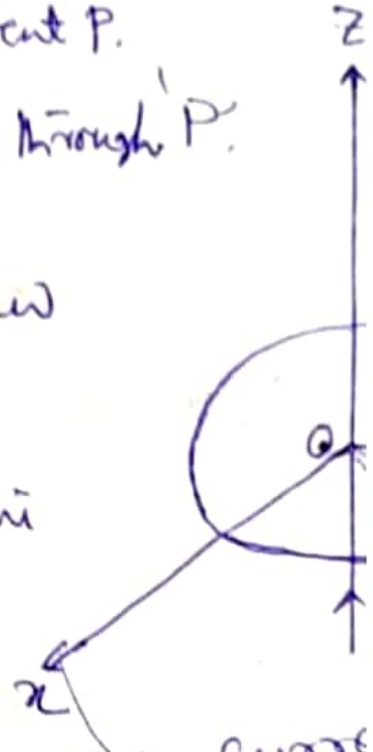
Let us consider an infinitely long filamentary current I along the z-axis.

Now to determine H at the point P.

We allow a closed path passes through P.

This path on which Ampere's law is to be applied is known as Amperian path (Analogous to the term Gaussian surface).



Since this path encloses the whole current

Ampere's law.

$$I = \oint H \cdot dl = \oint_{2\pi} H_\phi a_\phi \cdot \rho d$$

$$= H_\phi \int_0 \rho d\phi = H$$

$$\boxed{H_\phi = \frac{I}{2\pi\rho}}$$

(2) Infinitely Long Co-axial Transmissio

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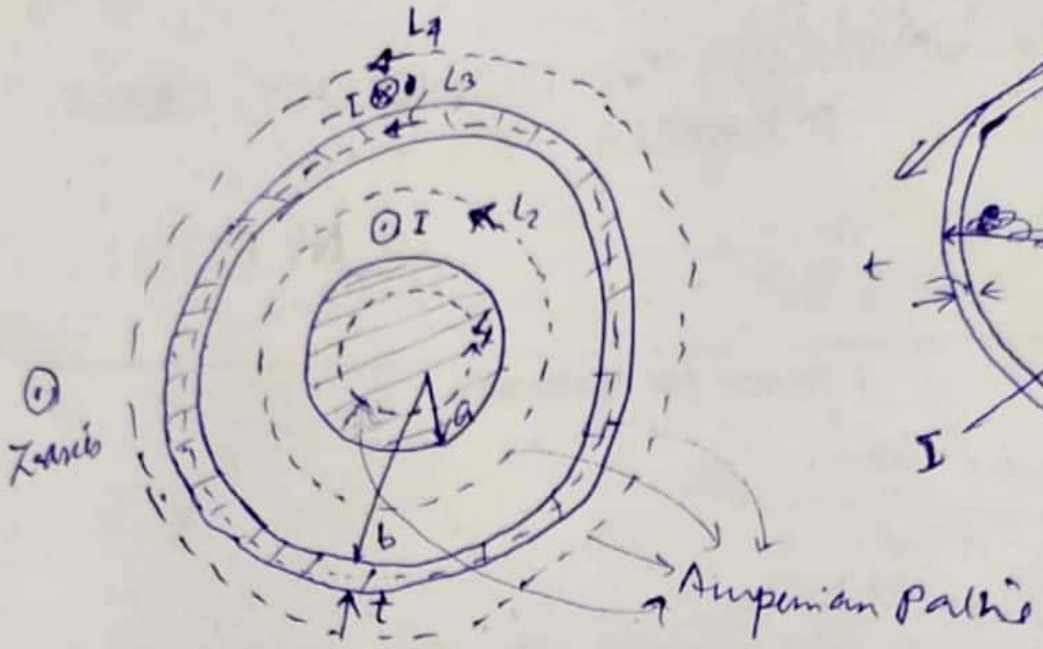

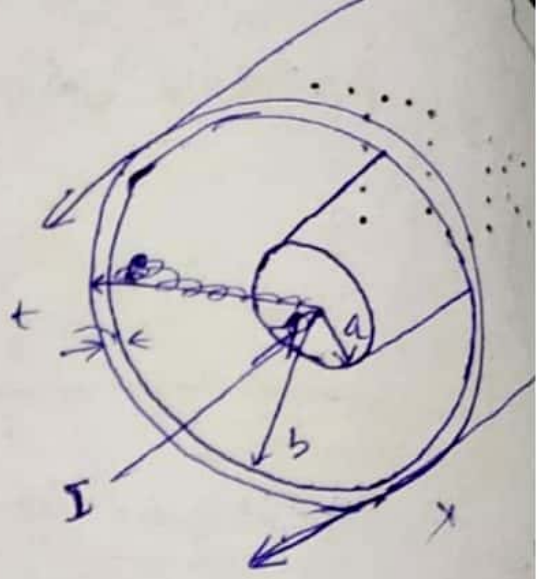

The inner conductor has a radius a and carries current I while the outer conductor has inner and thickness 't' and carries return current

we want to determine H everywhere assuming current is uniformly distributed in both Co. Since current distribution is symmetrical.

We apply Ampere's law along the Amperian path for each of the four possible regions.

$0 \leq \rho \leq a$

$a \leq \rho \leq b$

$b \leq \rho \leq b+t$

$\rho \geq b+t$.

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Case 1.

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The inner conductor has a radius
Current I while the outer conductor has inner
and thickness 't' and carries return current
we want to determine H everywhere assuming
Current is uniformly distributed in both conductors
Since current distribution is symmetrical.

we apply Ampere's law along the Amperian path
for each of the four possible regions.

$$0 \le \rho \le a$$
$$a \le \rho \le b$$
$$b \le \rho \le b+t$$
$$\rho \geq b+t.$$

Case 1.
For region $0 \le \rho \le a$, we apply Ampere's law to the

giving $\oint_L H \cdot dl = I_{enc} = \int J \cdot ds$ —— (1)

Since the current is uniformly distributed over the c
for J be the uniform current density in the solid c

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As the total current is I, then $J = \frac{I}{\pi a^2} a_z$,

$$ds = \rho\, d\phi\, d\rho\, a_z.$$

$$I_{enc} = \int J \cdot ds = \int \frac{I}{\pi a^2} a_z \cdot \rho\, d\phi\, d\rho\, a_z = \frac{I}{\pi a^2} \int_0^{\rho} \rho\, d\rho \int_0^{2\pi} d\phi = \frac{I}{\pi a^2} \cdot \pi \rho^2 = \frac{I}{a}$$

Hence from Eqn (1). $H_\phi \int dl = H_\phi \cdot 2\pi\rho = \frac{I\rho^2}{a^2}$

$$\boxed{H_\phi = \frac{I\rho}{2\pi a^2}}$$

General form ρ is varia

$\rightarrow$ at $\rho = a$ $H_\phi = \frac{I}{2\pi a}$

Case 2. For region $a \leq \rho \leq b$, we use path L_2 as the Amperian Pa

$$\oint_{L_2} H \cdot dl = I_{enc} = I = \text{Total Current}$$

$$H_\phi \cdot 2\pi\rho = I$$

$$\boxed{H_\phi = \frac{I}{2\pi\rho}}$$

where ρ is larger than the radi of the inner conductor which carries total current I.

Case 3. For region $b \leq \rho \leq b+t$, we use path L_3.

$$\oint H \cdot dl = H_\phi \cdot 2\pi\rho = I_{enc}.$$

Where $I_{enc} = I + \int J \cdot ds$ and J in this case is the current density of the outer conductor and is along $-a_z$.

$\boxed{(-z)}$

$$J = -\frac{I}{\pi\left[\pi(b+t)^2 - \pi b^2\right]} a_z$$

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Hence from ...

$$\boxed{H_\phi = \frac{I\rho}{2\pi a^2}}$$

Case 2. For region $a \le \rho \le b$, we use path L_2 as the Amperian

$$\oint_{L_2} H \cdot dl = I_{enc} = I = \text{Total Current}$$

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Case 3. For region $b \le \rho \le b+t$, we use path L_3.

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Where $I_{enc} = I + \int J \cdot ds$ and J in this case is the current density of the outer conductor and is along $-a_z$

$$J = -\frac{I}{\pi\left[(b+t)^2 - b^2\right]} a_z$$

$ds =$

Thus $$I_{enc} = I - \frac{I}{\pi\left[(b+t)^2 - b^2\right]} \int_{\phi=0}^{2\pi} \int_{b}^{\rho} \rho \, d\rho \, d\phi$$

$$= I - \frac{I\left[\rho^2 - b^2\right]}{t^2 + 2bt} = I\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$

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$$\boxed{H_\phi = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2-b^2}{t^2+2bt}\right]}$$

$= \frac{I}{2\pi\rho}\left[\frac{t^2+2bt-\rho^2+b^2}{t^2+2bt}\right]$

$=$

4. For a region $\rho \geq b+t$, we use path L_4, getting

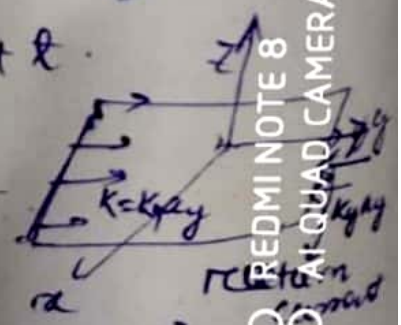
$$\oint_{L_4} H \cdot dl = I_{\text{inner cond}} + I_{\text{outer cond}} = I - I = 0$$

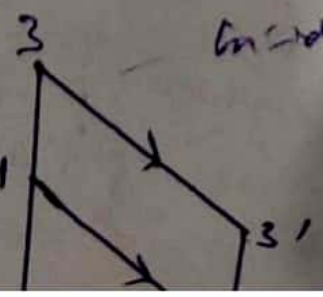
No current is enclosed.

$$\boxed{H_\phi = 0}$$

$$H = \begin{cases} \frac{I\rho}{2\pi a^2}\, a_\phi & 0 \leq \rho \leq a \\ \frac{I}{2\pi\rho}\, a_\phi & a \leq \rho \leq b \\ \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2-b^2}{t^2+2bt}\right] a_\phi & b \leq \rho \leq b+t \\ 0 & \rho \geq b+t \end{cases}$$

Inside





te Sheet of Current.

Consider a sheet of

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First statement ⇒

Let us consider the Ampere's circuital law in differential form without the displacement current density is

$$\nabla \times H = J.$$

If ΔS is an elementary surface, then taking dot product of both with ΔS.

($J \cdot \Delta S$ → is taken due to at a point we can consider; $\nabla \times H$ → also at a point)

$$(\nabla \times H) \cdot \Delta S = J \cdot \Delta S. \quad \text{—— (1)}$$

But According to Ampere's Circuital law in integral form without the displacement current is (due to at a point, the current passing)

$$\oint_c H \cdot dl = J \cdot \Delta S \quad \text{—— (2)}$$

where C is the closed path bounding ΔS.

Now we can compare eqn (1) and (2), we have

$$(\nabla \times H) \cdot \vec{\Delta S} = \oint_c H \cdot dl$$

$$(\nabla \times H) \cdot \Delta S a_n = \oint_c H \cdot dl$$

a_n is the unit vector normal to ΔS.

$$(\nabla \times H) \cdot a_n = \frac{\oint_c H \cdot dl}{\Delta S}$$

The max^m value of $(\nabla \times H) \cdot a_n$, can occur when a_n is oriented parallel to $\nabla \times H$, i.e when the surface ΔS is oriented normal to the current density vector J.

$$|\nabla \times H| = \left[\frac{\oint_c H \cdot dl}{\Delta S}\right]_{max}$$

Now to find out the dir^n, for this we

$$a_n = \frac{\nabla \times H}{|\nabla \times H|}$$

Conexness.

$$\nabla \times H = |\nabla \times H| a_n$$

$$\nabla \times H = \left[\frac{\oint_c H \cdot dl}{\Delta S}\right]_{max} a_n$$

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* So we can now define, the curl of H is an axial (or rotational) vector whose magnitude is the max^m circulation of H per unit area as the area tends to zero and whose dir^n is the normal dir^n of the area when the area is oriented so as to make the circulation max^m.

$$\nabla \times H = \text{curl } H = \left(\lim_{\Delta S \to 0} \frac{\oint_C H \cdot dl}{\Delta S} \right)_{max} a_n$$

where the area ΔS is bounded by the curve C. and a_n is the unit vector normal to the surface.

The curl of a vector field we can write in Cartesian Co-ordinate.

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} \qquad H = H_x a_x + H_y a_y + H_z a_z$$

$$\nabla \times H = \frac{1}{\rho} \begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}$$

$$\nabla \times H = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & r a_\theta & r \sin\theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & r H_\theta & r \sin\theta H_\phi \end{vmatrix}$$

<u>Physical Significance of the curl</u>

$$\nabla \times H = J$$

Curl deals with rotation. Consider a water whirl pool they are generally downwards. If a leaf is thrown into the pool it rotates and progress in the downward dir^n [illegible] of the curl of velocity vector is

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Piece of paper is released, it rotats and finally moves upwards. The dir^n of the vector 'J' $(\nabla \times H)$ is upwards.

The sign of the vector $(J = \nabla \times H)$ can be obtain using paddle wheel or curlmeter. In position.

① it rotates in anti clockwise dir^n, curl is taken as +ve
② If it rotats in clockwise dir^n, curl is taken as −ve
③ If it does not rotats, curl is zero.

$$\text{Curl } H = \frac{\text{Circulation}}{\text{Area}} = \frac{\text{work done}}{\text{Small area}} = \lim_{\Delta s \to 0} \frac{\oint F \cdot dl}{\Delta s}.$$

Stokes Theorem:

$$\oint_L H \cdot dl = \int_S (\nabla \times H) \cdot ds$$

Stokes theorem states that the circulation of a vector field H around a closed path L is equal to the surface integral of curl of H over the open surface S bounded by L provided that H and $\nabla \times H$ are continuous on S.

Proof: To prove the stokes theorem. Let us consider the surface S is subdivided into large no of cells. If the 1, 2, 3 .... Kth cell has surface area $\Delta S_1, \Delta S_2, \Delta S_3$ and is bounded by path $L_1, L_2, L_3 \ldots L_K$,

$$\oint A \cdot dl = \oint A \cdot dl + \int_{L_2} A \cdot dl + \int_{L_3} A \cdot dl + \ldots + \int A \cdot dl$$

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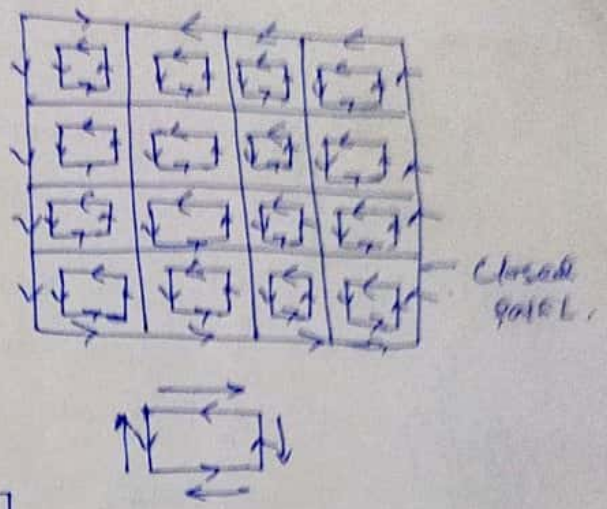
Here there is a cancellation on every interior path. So the sum of the line integral around L_k's is the same as the line integral around the bounding curve L. Therefore taking the limit of the right hand side as $\Delta S_k \to 0$.



$$\boxed{\oint_L H \cdot dl = \int_S (\nabla \times H) \cdot ds}$$

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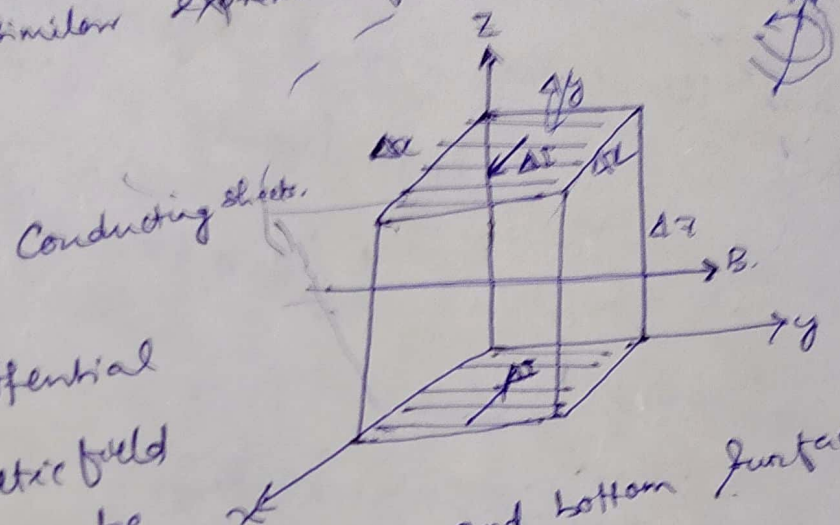
(Space for Writting)

Magnetic Energy :

we know the potential energy in an electrostatic field

$$W_e = \frac{1}{2}\int D\cdot E\, dv = \frac{1}{2}\int \epsilon E^2 dv$$

Now we can derive similar expression for the energy in a magnetostatic field.



Let us consider a differential volume in a magnetic field as shown. Let the volume be covered with conducting sheets at the top and bottom surface with current ΔI.

we assume that the whole region is filled with such differential volumes,

Each volume has an inductance

$$\Delta L = \frac{\Delta\Psi}{\Delta I} = \frac{B\,\Delta x\,\Delta z}{\Delta I}$$

$$= \frac{\mu H\,\Delta x\,\Delta z}{\Delta I} \quad \text{where } \Delta I = H\Delta y$$

$$\Delta W_m = \frac{1}{2}\Delta L\,\Delta I^2 = \frac{1}{2}\,\frac{\mu H\,\Delta x\,\Delta z}{\Delta I}\cdot\Delta I^2$$

$$= \frac{1}{2}\mu H\,\Delta x\,\Delta z\cdot H\Delta y$$

$$= \frac{1}{2}\mu H^2\,\Delta x\,\Delta y\,\Delta z = \frac{1}{2}\mu H^2$$

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$$\boxed{\Delta W_m = \frac{1}{2}\mu H^2 \Delta V}$$

The magnetostatic energy density w_m is defined as

$$w_m = \lim_{\Delta V \to 0} \frac{\Delta W_m}{\Delta V} = \frac{1}{2}\mu H^2$$

Hence $w_m = \frac{1}{2}\mu H^2 = \frac{1}{2} B \cdot H = \frac{B^2}{2\mu}$

$$\boxed{w_m = \frac{B^2}{2\mu}}$$

Thus the energy in a magnetostatic field in a dielectric

$$W_m = \int w_m dV$$

$$\boxed{W_m = \frac{1}{2}\int B \cdot H \, dV = \frac{1}{2}\int \mu H^2 dV}$$

~~Force on magnetic Materials~~

Magnetic Circuits :

we defined magnetomotive force (mmf) $\mathcal{F}$ as

$$\mathcal{F} = NI = \oint H \cdot dl$$

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